Invented Subtraction:

In your own way, solve the following contextual problem.

Alexander and Tai worked in the yard this afternoon for their uncle. They were asked to pick up the small rocks in the driveway. After a while, they decided to count the rocks they had collected, and they counted 201 small rocks. Their uncle wants to use some of the rocks in the backyard fountain, so they took some of their collection to the backyard. Their uncle helped count the rocks that they brought, and they used all 88 rocks in the fountain. How many rocks are left in the original pile?

Cassie's solution:

200 – 90 = 110 + 1 = 111 + 2 = 113

Alicia's solution:

88 + 2 = 90 + 10 = 100 + 100 = 200 + 1 = 201

Then, 2 + 10 = 12 + 100 = 112 + 1 = 113

John's solution:

```
  201
 - 88
 ----
   -7
   20
  100
 ----
```

100 + 20 – 7
= 120 – 7
= 113

Ways of Multiplying:

Marco is a first grader and demonstrates his own way of thinking about the problem 38*60.

> *4 times 6 is 24, so 40*60 is 2400. Then, 2*6 is 12, so it's 2400-120, which is 2280.*

What about Marco's solution gives clues to what he does know about multiplication? Is his thinking valid? Is his answer correct?

What properties of arithmetic did Marco use (knowingly or not) and where?

Tamar is a designated "special needs second grader." He used the following method to calculate 41*41:

> *Four 4s is 16, so four 40s is 160 and forty 40s is 1600. Then, forty-one 40s is another 40 added on, which is 1640. So, forty-one 41s is 41 more which is 1681.*

Explain why it makes sense for Tamar to solve the problem the way she does. What is the underlying understanding behind her solution strategy?

What properties of arithmetic did Tamar use (knowingly or not) and where?

Lindsay and Terrell have just begun to investigate fractions and percents in their third grade math class. The two students have been introduced to fraction notation, but they have not been given formal ways to perform operations with fractions.
Lindsay calculates two-fifths of 1260 in the following way:

> *First, I find half of 1260, which is 630. Then, I subtract one-tenth of 1260, which is 126 from 630, and this gives the answer of 504.*

Terrell is in the same class and he calculates two-fifths of 1260 this way:

> *First, I multiplied 1260 by 2 to get 2520. Then, I multiplied 2520 by 2 to get 5040. Then, I just simply divided by 10 to get 504.*

What are the underlying ideas behind the two strategies? Did the students get the correct answer? How do you know? Which method is more efficient? Why?

Division can be easily understood as fair sharing!

As part of their holiday celebration, Ms. Miller's class is carving a pumpkin. She puts the seeds on a baking sheet, and the students in the class help her count a total of 358 pumpkin seeds. She wants the students to make an art project out of the seeds, and decides that the twenty-five students in her class will need an equal amount of pumpkin seeds to complete their picture. Trying to embed realistic experiences into her math class, she asks her second-grade students to help her figure out how many seeds each student should receive.

Lucy's work is shown below. Explain why her thinking and solution strategy make sense.

$25 \times 2 = 50$

$7 \times 50 = 350$

$7 \times 2 = 14$

14 R8

Merlin's solution to the same problem is shown below. What is he doing that Lucy is not? What does this solution strategy tell you about Merlin's understanding of division?

$10 \times 25 = 250$

$250 + 50 = 300$

$300 + 50 = 350$

$10 + 2 + 2 = 14$

remainder 8

Haylee's solution is demonstrated for the class as well. What does her solution strategy demonstrate about her understanding? How is it different or similar to the other solution methods?

$25 \times 2 = 50$

$50 \times 2 = 100$

$100 \times 3 = 300$

$300 + 50 = 350$

8 left

$2 \times 2 \times 3 = 12$

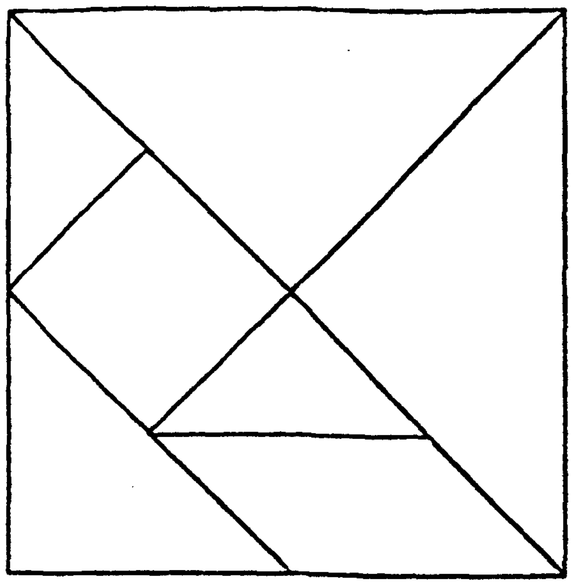
$12 + 2 = 14$

with 8 leftover

Nobody knows how old Aunt Helen is but she gave a few hints. She had passed $\frac{1}{20}$ of her life before she started school. She spent $\frac{3}{20}$ of her life in school; She worked for $\frac{1}{10}$ of her life before she got married. She was married for $\frac{2}{5}$ of her life. Her husband died after $\frac{7}{10}$ of her life. From reading Uncle Harry's gravestone you find out that she has been a widow for 24 years. How old is Aunt Helen?

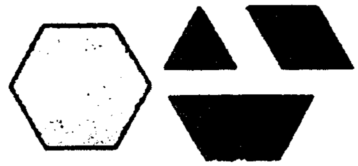
TANGRAM CAKES

If this Tangram Cake costs **$10.00** how much will each piece cost?



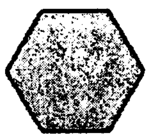
Name of Piece	Fraction of Cake	Cost

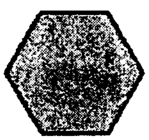
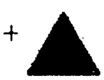
Cynthia Lanius

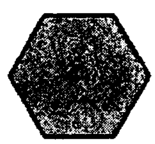


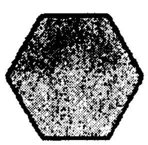
More Fun Fractions

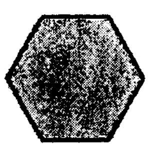
Let's do some *really* fun ones.

1. If  +  = 1, what is  ?

2. If  +  = 1, what is  +  ?

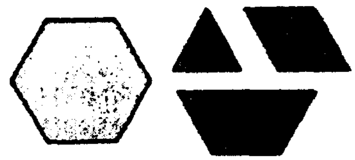
3. If  +  = 1, what is  +  ?

4. If  +  = 1, what is  ?

5. If  -  = 1, what is  +  ?

[redacted]

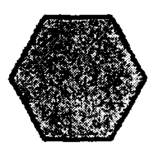
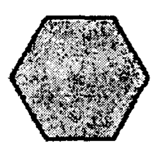
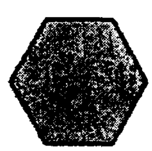
Cynthia Lanius



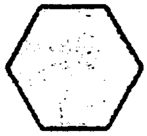
No Matter What Shape Your Fractions are In

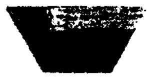
Determining the Relations

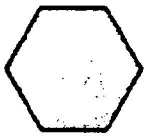
Use the [redacted] pattern blocks to answer the following questions.

1. How many  are in  ?
2. How many  are in  ?
3. How many  are in  ?
4. How many  are in  ?
5. How many  are in  ?
6. How many  are in  ?

Based on these relations,

7. If  = 1,  = ___ .

8. If  = 1,  = ___ .

9. If  = 1,  = ___ .

10. If  = 1,  = ___ .

Fractions of Rods

A. Place a brown rod in the top row of the outline shown.

Find all the ways to make one-color trains that are as long as the brown rod. Place them in the outline.

brown	
purple	purple

If the brown rod is 1,

- find the value of the purple rod. How do you know?
- find the rod that is 1/4 of the brown rod. How do you know?
- find the rod that is 1/8 of the brown rod. How do you know?
- show 1/2 = 2/4 = 4/8 with the rods. Explain your reasoning.

B. Place an orange-red rod in the top row of the outline shown.
Find all the ways to make one-color trains that are as long as the orange-red rod. Place them in the outline.

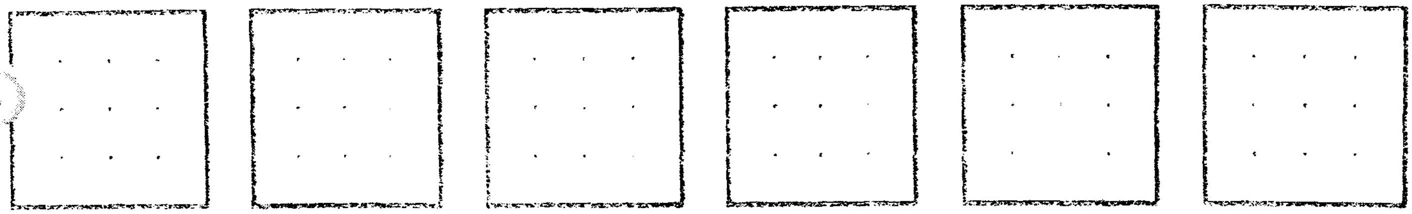
Note: Tape an orange rod and a red rod together to create an orange-red rod.

If the orange-red rod is 1,

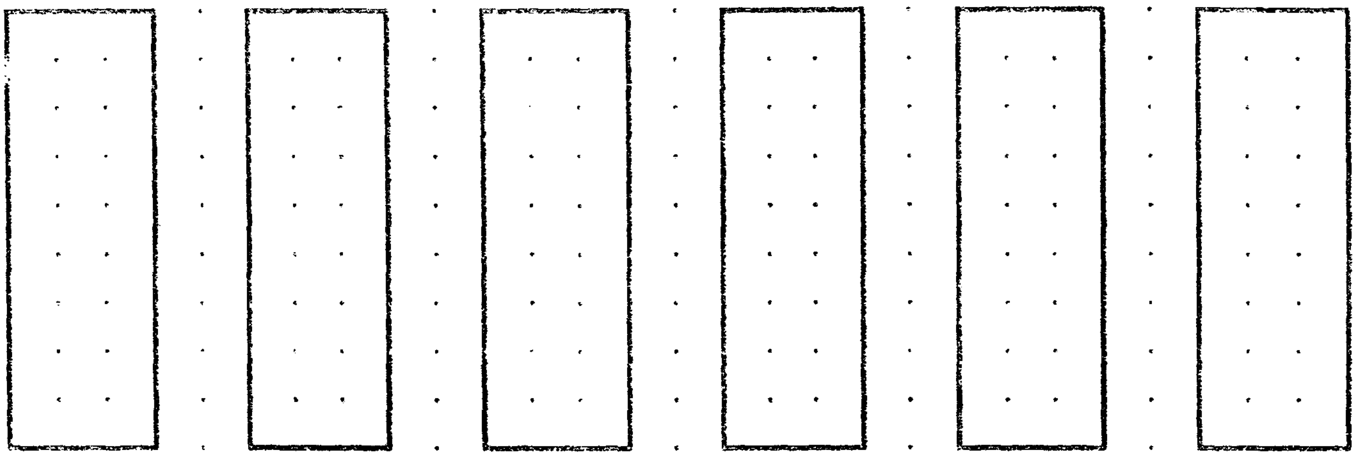
- find the rod that is 1/2 of the orange-red rod. How do you know?
- find the rod that is 1/4 of the orange-red rod. How do you know?
- find the value of every rod you used. How do you know?
- show 1/2 = 2/4 = 3/6 with the rods. Explain your reasoning.

C. Compare your results when the brown rod is 1 to the results when the orange-red rod is 1. Write about your findings.

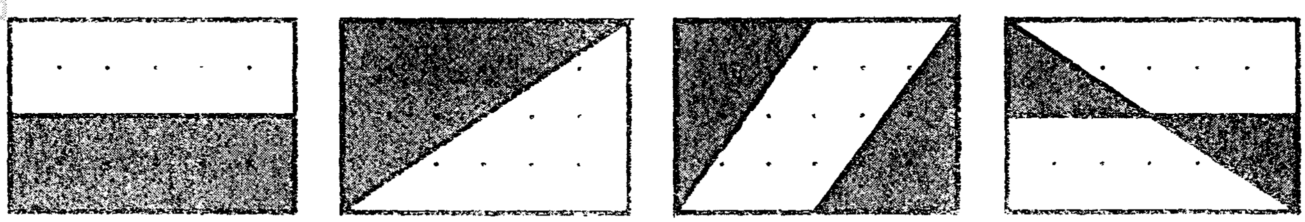
A. Shade in 1/2 of the square in at least six ways.



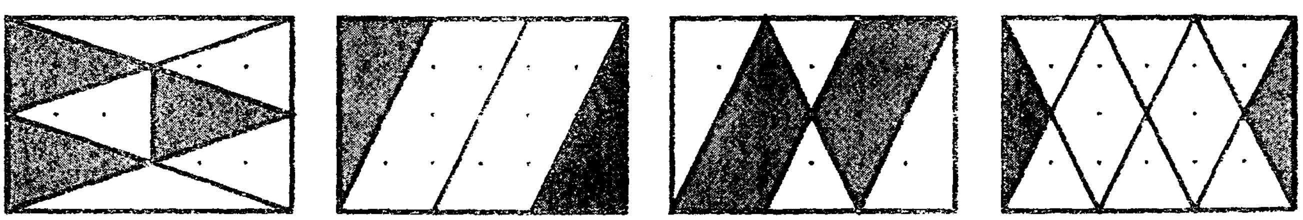
B. Shade in 1/3 of the rectangle in at least six ways.



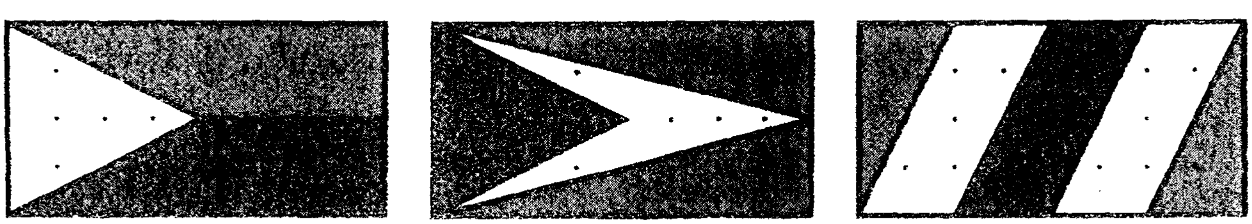
C. Determine what fraction of each flag is shaded.



1. 2. 3. 4.

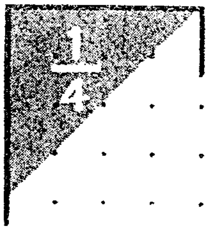


5. 6. 7. 8.



9. 10. 11.

A. Draw the whole rectangle from which the shaded region was made.

1.


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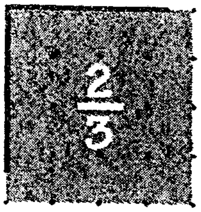
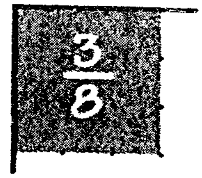
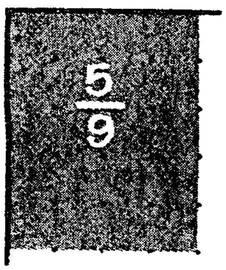
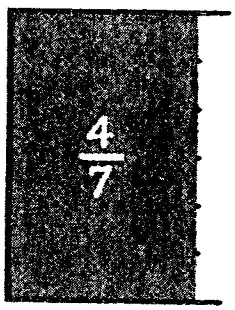


3.


4.


5.


6.


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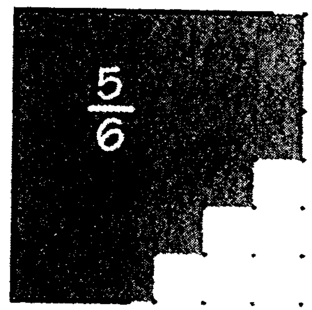

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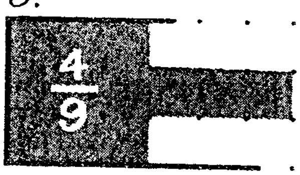
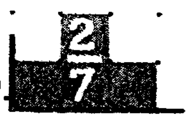
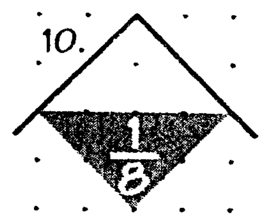
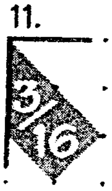
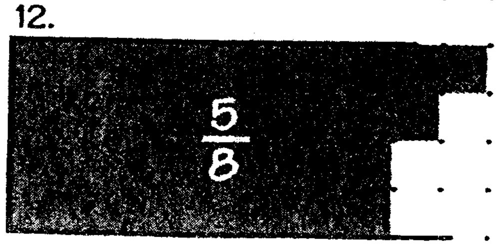


9.


10.


11.


12.


Name ______________________________

EDEC 330
Week Four
Developing Fraction Concepts Station IV – Cuisenaire Rods

For this activity, you will need:

- One set of Cuisenaire rods per group member
- Tape
- Pencil or Pen
- Your brain

1. From your set of rods, select a brown rod. In this first problem, the brown rod represents one whole unit. Find all of the ways (there are four) to make one-color trains of rods that are the same length as one brown rod.

   Once you have all four of the one-color trains, consider the following questions. If the brown rod represents one whole, what is the numerical value of the purple rod? How do you know? Find the rod that is $\frac{1}{4}$ of the brown rod. How do you know? Find the rod that is $\frac{1}{8}$ of the brown rod. How do you know? Show that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ with the rods. Explain your reasoning.

2. Use the tape to secure an orange rod and a red rod together to create an orange-red rod. Find all of the ways (I'm not telling how many there are) to make one-color trains of rods that are the same length as the orange-red combination rod. Record your responses on a separate sheet of paper.

   Find the rod that is $\frac{1}{2}$ of the orange-red rod. How do you know? Find the rod that is $\frac{1}{4}$ of the orange-red rod. How do you know? Find the numerical value of every rod that you used in this second situation. How do you know? Show that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$ with the rods. Explain your reasoning!

3. Compare your results when the brown rod is one to the results when the orange-red rod is one. What conclusions can you draw about fractions and fraction concepts from these two situations?

4. Using the conditions for when the brown rod is one, use the rods to demonstrate why the number sentence $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ makes sense. Keep a written record of your thinking on a separate sheet of paper.

5. Using the conditions for when the orange-red rod is one, use the rods to demonstrate why the number sentence why the number sentence $\frac{3}{4}-\frac{1}{3}=\frac{5}{12}$ makes sense. Keep a written record of your thinking on a separate sheet of paper.

EDEC 330
Week Four
Developing Fraction Concepts Station V – Fraction Circles

For this station, you will need:
- One set of fraction circles for each group member
- One spinner
- The empty top and empty bottom of your fraction circle container
- Pencil or Pen
- Your brain

Game I:
In this game, the object is to completely fill the top or bottom of your fraction circle container with fraction circle pieces. The pieces may NOT overlap when placed into your fraction circle container.

How to play:
- Place all of the fraction circle pieces in a pile in front of you. Turn the empty top and empty bottom of your fraction circle container up so that you can place pieces inside the cases to make circles.
- Each player, in turn (oldest person in the group goes first), will spin the spinner and take a piece of the color shown from their piece pile. Place this piece in either part of the fraction circle container. Once you place a piece, it cannot be moved.
- On your next turn, you will spin again. Put the fraction piece shown with the one already in the container or start a new circle in the other part of the container. Pieces cannot overlap.
- Play continues around the group.
- Each time you complete "one whole" circle, record the color combinations that were used to form the whole, place the pieces back into the pile, and give yourself one point.
- Each time your spinner shows a piece that does not fit in either of your container top or bottom, or if there are no more colors left, record an X.
- The game is over when one player has recorded three X's. The highest point total wins.
- Play several times before considering these questions: Is it possible to score a point in four spins? Less than four spins? WHY? What is the lowest possible score you could get? What is the greatest possible score?

Game II:

With your results from Game I and with the collective thinking of your group, consider the following challenge:

In how many different ways can you use the fraction circle pieces to make circles of one or more colors? Find and record as many ways as you can.

one color: | two color: | three colors:

four (or more) colors

Convince me that $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$

Algebra in the Elementary Grades? Absolutely!

By Marilyn Burns

Algebra in the Elementary Grades? Absolutely!

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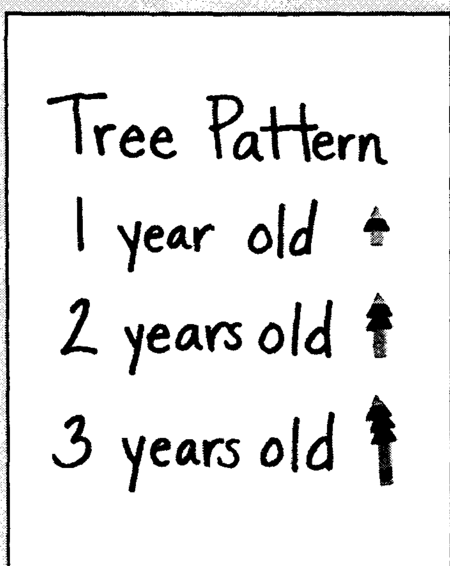
THE LESSON



1 To begin the lesson, I used pattern blocks to build a tree and show how it grows. "This is a young tree that's only one year old," I explained as I put a small loop of masking tape on the back of each of three pattern blocks and stuck them on the board (one orange square, one red trapezoid, and one green triangle). I wrote *1 year old* to the left of the blocks. I then wrote *2 years old* underneath and constructed another tree, this time with five blocks (two squares and two trapezoids plus one green square). Jamaal volunteered how to build a three-year-old tree; I wrote *3 years old* on the board and taped up the seven blocks he identified.

2 Mylla was eager to extend the pattern. "Next year it will have four and four, and the triangle on top," she said. I sketched Mylla's idea on the board. I then drew a T-chart and labeled the columns *Years Old* and *Blocks*. I wrote a *1* in the *Years Old* column and asked, "How many blocks did I use for a one-year-old tree?"

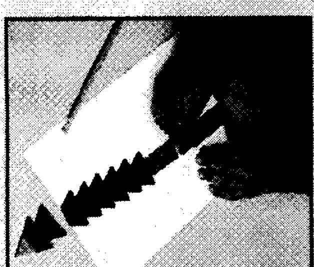
"Three," the children responded. I recorded a 3 in the *Blocks* column. With the children's help, I filled in the number of blocks for two-, three-, and four-year-old trees and then, using dots to indicate that I was skipping numbers, wrote 10 in the left column with a question mark next to it, and, farther down, 25. Finally, I wrote on the board two problems for the students to solve: *How many blocks are in a 10-year-old tree? How many blocks are in a 25-year-old tree?* "You can use the blocks to help you," I explained to the students. "Also, record on a T-chart, look for patterns, and write about how you solved the problems."

PHOTOS: BOB ADLER

While most of us first encountered algebra and the world of x's and y's when we reached high school, developing algebraic thinking is a top priority in today's elementary math curriculum. Algebra is now second in importance after number and operations, even for elementary students. This is a fairly recent requirement, and the NCTM *Principles and Standards for School Mathematics* explains why: "By viewing algebra as a strand in the curriculum from prekindergarten on, teachers can help students build a solid foundation of understanding and experiences as preparation for more sophisticated work in algebra in the middle grades and high school."

However, many elementary teachers are not comfortable with their own memories of algebra, much less with teaching it to their young students. In this article, Marilyn Burns invites you into a second-grade class in Emeryville, California, and gives you a step-by-step look at an algebra lesson. Not only does the lesson give the children experience with the important algebraic ideas of interpreting, extending, and representing a growth pattern, it also supports the children's learning about number and geometry.

3 As the children got to work, I circulated and gave help as needed. Although most of the students got started on their own, Eliyah and Amanjot (above) needed help. I sat with them and explained again what they were to do, then left them to work on their own.



4 Most students built the trees with blocks. Some, like Pierre, recorded by carefully tracing or drawing the blocks.

Other students, such as Carla, didn't draw or trace the blocks, but simply recorded numerically after building each tree.

A few students, like Christian, didn't need to use the blocks or record on a T-chart, but instead solved the problem in their heads.

5 As I circulated, I observed and talked with children to learn more about how they were approaching the problem and reasoning. I asked Yelmy (above) to explain to me how she knew that the four-year-old tree she built was correct. Our conversation didn't distract Paul from concentrating on tracing blocks. *(Continued on page 26)*

Early Algebraic Concepts

The following algebraic concepts are important in the elementary grades.

- **PATTERNS**: Creating, recognizing, extending, and generalizing growth patterns
- **EQUIVALENCE**: Understanding the equal sign as an indication that quantities have the same value, not as a signal to write the answer
- **VARIABLES**: Using symbols to describe a relationship between two quantities (such as the age and number of blocks for trees), to stand for unknown quantities, to represent mathematical properties (such as a + b = b + a), and in formulas
- **GRAPHING**: Using pairs of numbers to plot points, thus learning another way to represent a relationship between two quantities

What About Older Students?

The lesson as described can be extended for older students. While the focus for younger students is to represent patterns concretely with materials, numerically on T-charts, and verbally, older students can also describe the pattern of growth using algebraic symbols. For any age tree, the total number of blocks is equal to twice its age (once for the orange squares and once for the red trapezoids) plus one more block (the green triangle at the top).

Using Δ to represent the total number of blocks and ○ for the tree's age, you can represent this relationship as

$$\Delta = \circ + \circ + 1 \text{ or } \Delta = 2 \times \circ + 1.$$

Students should also have experience using letters for variables. For example, the relationship can be expressed as

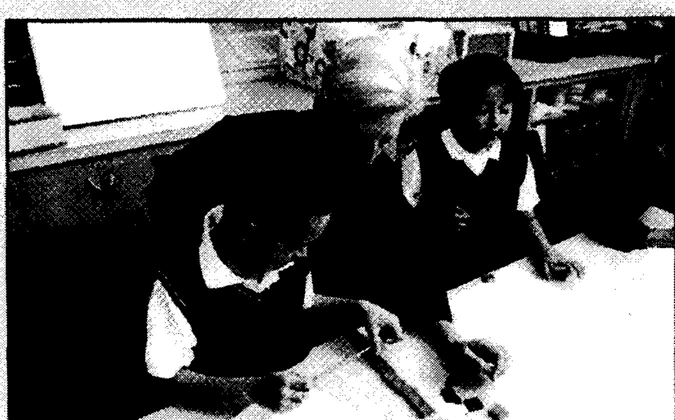
$$y = x + x + 1 \text{ or } y = 2x + 1.$$

Relationships can also be represented as graphs by plotting the pairs of numbers from the T-chart and investigating the pattern formed by the points. For the tree pattern, the points all lie on a diagonal line. The second graders in the lesson shown below had learned to plot points and identify their coordinates, but they hadn't yet used this skill as another way to represent patterns.

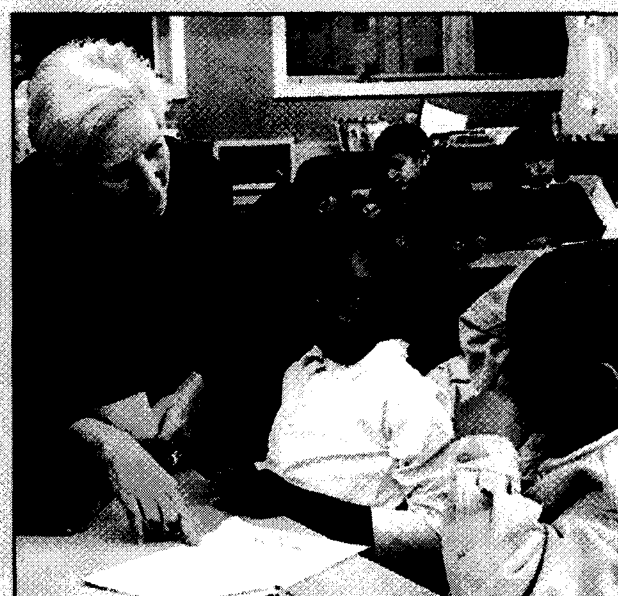
THE LESSON

(Continued from page 25)

6 Next, I helped Laura (left) confirm that what she had done so far matched what was recorded on the board.



7 Then, Pierre (above) showed me how he counted the blocks to figure how many were in a 10-year-old tree.

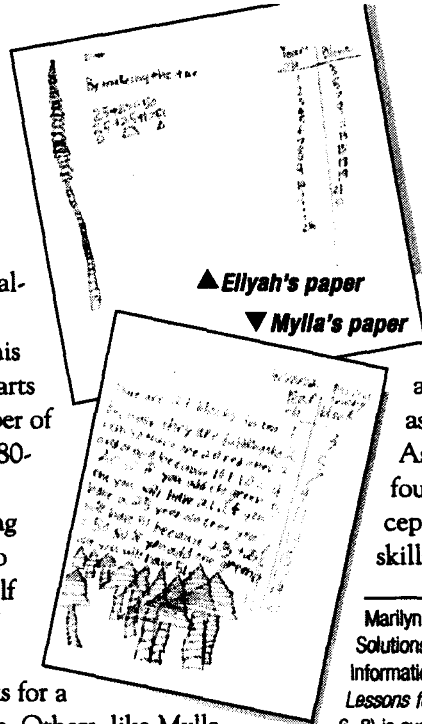


8 Bismilla (above) wasn't sure about how Christian had solved the problem without using the blocks, so I asked Christian to explain to her what he had done. He gave an example and then generalized. He said, "I know because ten plus ten is twenty, and one more is twenty-one. They're all like that. You go the number plus the number plus one more." Christian's explanation was mathematically sophisticated for his age. *(Continued on page 28)*

Children Show Their Work

Organizing their work is a valuable learning experience for children, and for that reason I gave the children blank paper rather than a prepared worksheet with a T-chart already drawn and space for them to draw and write. When children have to take responsibility for representing their work, they focus on making sense of the problem, not merely on filling in the answers.

As is typical, the children's papers differed. Some, like Paul and Amanjot, focused on tracing the blocks and didn't have time to figure out the answers to the problems. Other chose not to trace or draw the trees, but worked numerically; Robert, for example, filled his paper with T-charts listing the number of blocks up to an 80-year-old tree. Christian, finding the problems too easy, gave himself the challenge of figuring out the number of blocks for a 113-year-old tree. Others, like Mylla, Martell, and Nicole, included drawings, T-charts, and explanations about how they solved the problems.



▲ *Ellyah's paper*

▼ *Mylla's paper*

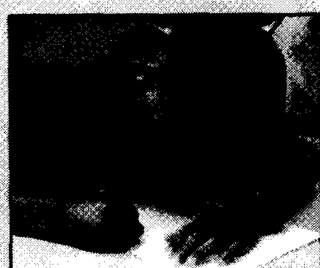
Children's papers are extremely useful for assessing their progress. As students build a solid foundation in algebraic concepts, they will gain valuable skills for future success. ■

Marilyn Burns is the founder of Math Solutions Inservice and Publications. Information about her first three-book series, *Lessons for Algebraic Thinking* (K–2, 3–5, and 6–8) is available at www.mathsolutions.com

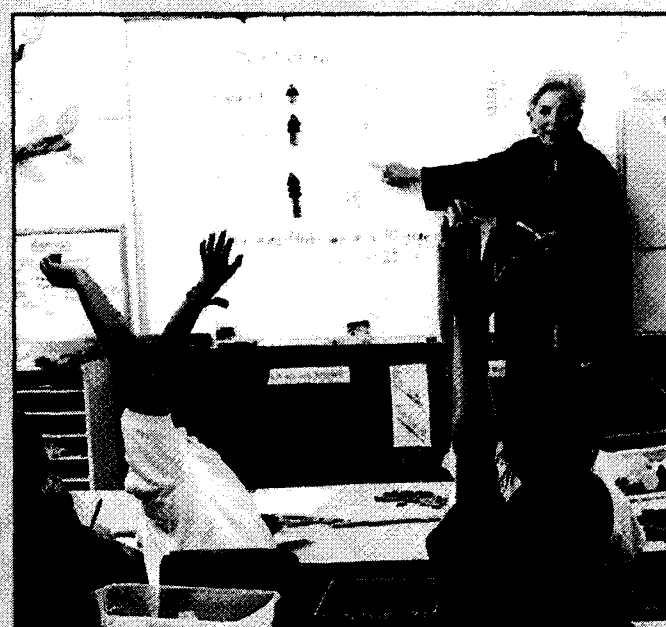
THE LESSON

(Continued from page 26)

9 I observed Robert (right) continue the pattern in his T-chart up to an 80-year-old tree.



▼ Then, Jamaal (below) asked me for help. He was concerned because he knew that a 25-year-old tree should have 51 blocks, but on his T-chart, he had written 49 blocks. He had misaligned some numbers, and I helped him find his error.



10 After the children completed their work on the assignment, I began a class discussion. The children were eager to report how many blocks were needed for the 10- and 25-year-old trees. I also asked them to explain the patterns they noticed and their methods for finding the solutions.

Better Mathematics Through Literacy
Day Five – Warm-up Problem

Finding patterns

In each of the following sequences, there is a pattern. Your task is to determine what pattern was used to generate the sequence (not all patterns are numeric!), state the pattern in your own words, and then, give the next three items in the sequence.

1. 1, 10, 100, 1000...

2. 0, 10, 21, 33, 46, 60...

3. 1, 4, 9, 16, 25, 36...

4. $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}...$

5. 180, 360, 540, 720...

6. A, B, D, G, K...

7. J, F, M, A, M...

8. Montreal, Moscow, Los Angeles, Seoul, Barcelona...

9. 1, 1, 2, 3, 5, 8...