Representation Standard for Grades Pre-K–2

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Young students use many varied representations to build new understandings and express mathematical ideas. Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Teachers should analyze students' representations and carefully listen to their discussions to gain insights into the development of mathematical thinking and to enable them to provide support as students connect their languages to the conventional language of mathematics. The goals of the Communication Standard are closely linked with those of this Standard, with each set contributing to and supporting the other.

Students in prekindergarten through grade 2 represent their thoughts about, and understanding of, mathematical ideas through oral and written language, physical gestures, drawings, and invented and conventional symbols (Edwards, Gandini, and Forman 1993). These representations are methods for communicating as well as powerful tools for thinking. The process of linking different representations, including technological ones, deepens students' understanding of mathematics because of the connections they make between ideas and the ways the ideas can be expressed. Teachers can gain insight into students' thinking and their grasp of mathematical concepts by examining, questioning, and interpreting their representations. Although a striking aspect of children's mathematical development in the pre-K–2 years is their growth in using standard mathematical symbols, teachers at this level should encourage students to use multiple representations, and they should assess the level of mathematical understanding conveyed by those representations.

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What should representation look like in prekindergarten through grade 2?

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Young students represent their mathematical ideas and procedures in many ways. They use physical objects such as their own fingers, natural language, drawings, diagrams, physical gestures, and symbols. Through interactions with these representations, other students, and the teacher, students develop their own mental images of mathematical ideas. Although the representations that children use may not be those traditionally used by adults, students' representations provide a record of » their efforts to understand mathematics and also make their understanding available to others.

Representations make mathematical ideas more concrete and available for reflection. Students can represent ideas with objects that can be moved and rearranged. Such concrete representations lay the foundation for the later use of symbols. Students' representations are often insightful and many times resemble more-conventional representations. For example, a second grader working with place-value mats and base-ten blocks can represent 103. The student might point to the blocks and tap at the empty column, explaining, "One hundred, (tap), three." The tap helps the student connect the zero with the empty tens column.

The following account of a lesson, drawn from a classroom experience, illustrates that what children do and say as they find answers and represent their thinking gives teachers information about their levels of understanding.

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> When a first-grade teacher read *Rooster's Off to See the World* (Carle 1971), the students' representations of the number of animals going off to see the world varied (see fig. 4.34). Two cats, then three frogs, four turtles, and five fish joined the rooster for a total of fifteen animals. To find how many went on the trip, some students drew the animals and numbered them. Two students modeled the animals with counters, counted, and wrote "15" on their papers. Other students used more traditional notations, although their representations revealed different ways of thinking. One student declared the answer to be zero, because all the animals had gone home when it got dark. »

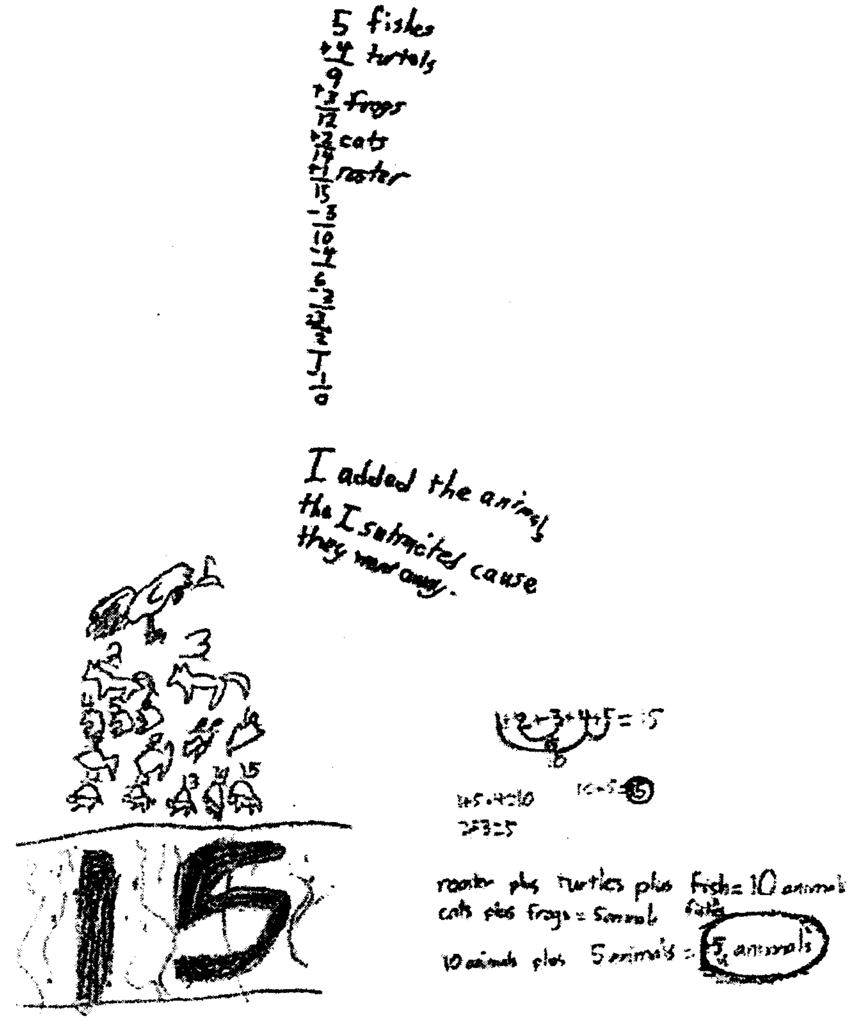

Fig. **4.34.** Three students' representations for the numbers of animals that went off to see the world

The teacher was puzzled, however, by the student whose answer was 21 (see fig. 4.35). The teacher asked the student to explain. The student responded that she had noticed that there were fireflies in the story on the page where the animals decided to turn around and go home. She couldn't count how many, but she thought there were six because that was the pattern, so she drew the animals and added. The teacher asked about the list at the top right of the student's paper, and she responded that she had made the tallies to show how many, and there were 21.

Fig. **4.35.** A student's representation of the number of animals that went off to see the world

Representations help students recognize the common mathematical nature of different situations. Students might represent the following three scenarios by writing 5 – 3 = 2. The first problem is to determine the number of objects left after three objects have been taken from a collection of five. The second problem is, How much taller is a tower of five cubes than a tower of three cubes? The third problem asks the number of balls that must be put into a box if it is to have five balls and there are already three in the box. Students could also represent the situations as 3 + □ = 5. Seeing similarities in the ways to represent different situations is an important step toward abstraction.

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Students use representations to organize their thinking. Representations can carry some of the burden of remembering by letting students record intermediate steps in a process. For example, a student trying to find the number of wheels in four bicycles and three tricycles drew the picture shown in figure 4.36. In the first row, the student represented the number of wheels on the bicycles and in the second, the number of » wheels on the tricycles. Thus the student was able to compute the sums separately and add them together. The representation served as a placeholder for thoughts that were not yet internalized.

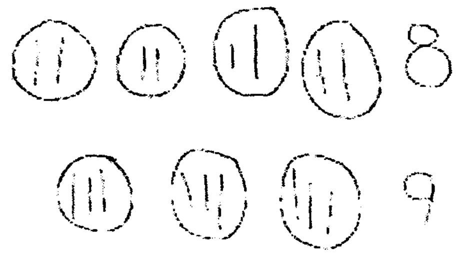

Fig. **4.36.** A student's representation of the number of wheels on four bicycles and three tricycles (Adapted from Flores [1997, p. 86.])

Understanding and using mathematical concepts and procedures is enhanced when students can translate between different representations of the same idea. In doing so, students appreciate that some representations highlight features of the problem in a better way or make it easier to understand certain properties. For example, a student who represents three groups of four squares as an array and uses skip-counting (4, 8, 12), repeated addition (4 + 4 + 4), and oral language to describe the representation as three rows of four squares is laying the groundwork for understanding multiplication and its properties as well as the area of a rectangle.

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What should be the teacher's role in developing representation in prekindergarten through grade 2?

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged, supported, and accepted by their peers and adults. Teachers should guide students to develop and use multiple representations effectively. Students will thus develop their own perceptions, create their own evidence, structure their own analytical processes, and become confident and competent in their use of mathematics.

Teachers at this level need to listen to what students say, thoughtfully observe their mathematical activities, analyze their recordings, and reflect on the implications of the observations and analyses. Using representations helps students remember what they did and explain their reasoning. Representations furnish a record of students' thinking that shows both the answer and the process, and they assist teachers in formulating questions that can help students reflect on their processes and products and advance their understanding of concepts and procedures. The information gathered from these multiple sources makes possible a clearer assessment of what students understand and what mathematical ideas are still developing.

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Students should be encouraged to share their different representations to help them consider other perspectives and ways of explaining their thinking. Teachers can model conventional ways of representing mathematical situations, but it is important for students to use representations that are meaningful to them. Transitions to conventional » notations should be connected to the methods and thinking of the students. For example, when students use blocks or mental computation to solve a problem like "Find the sum of 17 + 25," they frequently add the tens first. Teachers can write the intermediate steps for students as 10 + 20 = 30, 7 + 5 = 12, and 30 + 12 = 42. Students should see their method recorded both horizontally and vertically and should develop their own ways of keeping track of their work that are clear to them (see fig. 4.37).

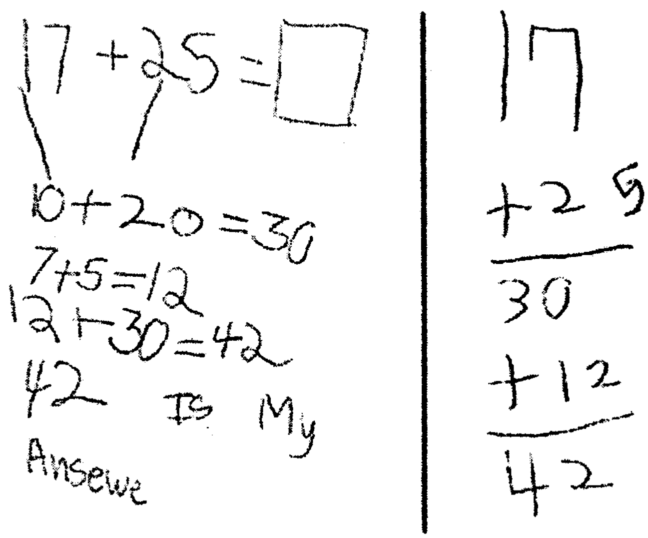

Fig. **4.37.** Recording a method to find 17 + 25 in two different ways

Through class discussions of students' ways of thinking and recording, teachers can lay foundations for students' understanding of conventional ways of representing the process of adding numbers. Equally important, students' work and conversations about their representations can reveal the extent to which they understand their use of symbols.

Written work often does not reveal a student's entire thinking, as the following hypothetical story about Armando demonstrates:

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> Armando does not show marks to cross out any digits or write a small "1" to represent "borrowing" with paper-and-pencil subtraction, but he consistently writes correct answers (see fig. 4.38). When the teacher probes, Armando explains that he has learned a different way to subtract at home. The teacher asks him to explain his method. "From 8 to 14—that is 6, and we need to add 1

to the 2 because we used 14 instead of 4." He writes 6 in the units place and continues, "From 3 to 7 is 4," and he writes 4 in the tens column. The teacher rephrases the second part of the method, emphasizing place value: "So, you add 10 to the 20 and then subtract 70 – 30." Realizing that the method is based on a property the class has recently discussed, that the same number can be added to both terms » of a difference and the result does not change, she invites the class to talk about the process Armando is using. After some discussion, one student explains, "You are adding 10 to 74 because you really did 14 – 8, and you also added 10 to 28 because you did 70 – 30. So the answer is the same."

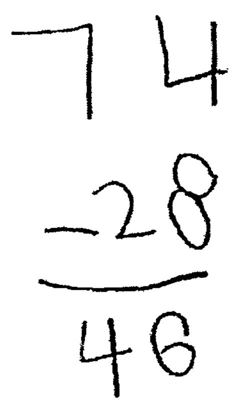

Fig. **4.38.** Finding 74 – 28 without "borrowing"

Teachers should help students understand that representations are tools to model and interpret phenomena of a mathematical nature that are found in different contexts. Teachers should help students represent aspects of situations in mathematical terms, possibly by using more than one representation. Technology may help students who are challenged by oral or written communication find greater success. The processing schema required in some computer programs can aid students in showing what they know. For example, when a student changes the representation of a number with base-ten blocks on the screen, the computer shows how the corresponding symbols change.

It is important that teachers realize and teach students that any representations, not only those created by students, are subject to multiple interpretations. Drawings, charts, graphs, and diagrams, for instance, can be read in different ways. Therefore, teachers should not assume that students understand a diagram or equation the same way adults do. Communicating the intended meaning and using alternative representations can enhance understanding by students and teachers alike.