Reasoning and Proof Standard for Grades Pre-K–2

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Young students are just forming their store of mathematical knowledge, but even the youngest can reason from their own experiences (Bransford, Brown, and Cocking 1999). Although young children are working from a small knowledge base, their logical reasoning begins before school and is continually modified by their experiences. Teachers should maintain an environment that respects, nurtures, and encourages students so that they do not give up their belief that the world, including mathematics, is supposed to make sense.

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Although they have yet to develop all the tools used in mathematical reasoning, young students have their own ways of finding mathematical results and convincing themselves that they are true. Two important elements of reasoning for students in the early grades are pattern-recognition and classification skills. They use a combination of ways of justifying their answers—perception, empirical evidence, and short chains of deductive reasoning grounded in previously accepted facts. They make conjectures and reach conclusions that are logical and defensible from their perspective. Even when they are struggling, their responses reveal the sense they are making of mathematical situations.

Young students naturally generalize from examples (Carpenter and Levi 1999), so teachers should guide them to use examples and counterexamples to test whether their generalizations are appropriate. By the end of second grade, students should be using this method for testing their conjectures and those of others.

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What should reasoning and proof look like in

prekindergarten through grade 2?

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The ability to reason systematically and carefully develops when students are encouraged to make conjectures, are given time to search for evidence to prove or disprove them, and are expected to explain and justify their ideas. In the beginning, perception may be the predominant method of determining truth: nine markers spread far apart may be seen as "more" than eleven markers placed close together. Later, as students develop their mathematical tools, they should use empirical approaches such as matching the collections, which leads to the use of more-abstract methods such as counting to compare the collections. Maturity, experiences, and increased mathematical knowledge together promote the development of reasoning throughout the early years. »

Creating and describing patterns offer important opportunities for students to make conjectures and give reasons for their validity, as the following episode drawn from classroom experience demonstrates.

> The student who created the pattern shown in figure 4.27 proudly announced to her teacher that she had made four patterns in one. "Look," she said, "there's triangle, triangle, circle, circle, square, square. That's one pattern. Then there's small, large, small, large, small, large. That's the second pattern. Then there's thin, thick, thin, thick, thin, thick. That's the third pattern. The fourth pattern is blue, blue, red, red, yellow, yellow."
>
> Her friend studied the row of blocks and then said, "I think there are just two patterns. See, the shapes and colors are an AABBCC pattern. The sizes are an ABABAB pattern. Thick and thin is an ABABAB pattern, too. So you really only have two different patterns." The first student considered her friend's argument and replied, "I guess you're right—but so am I!"

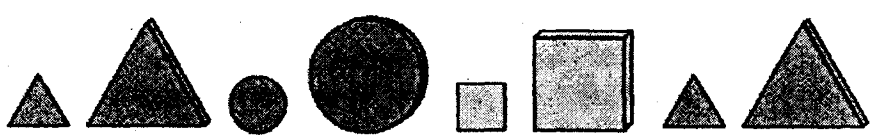
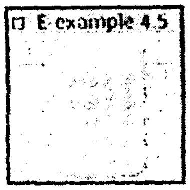


Fig. **4.27.** Four patterns in one

Being able to explain one's thinking by stating reasons is an important skill for formal reasoning that begins at this level.



Calculators and Hundred Boards (Part 2)

Finding patterns on a hundred board allows students to link visual patterns with number patterns and to make and investigate conjectures. Teachers extend students' thinking by probing beyond their initial observations. Students frequently describe the changes in numbers or the visual patterns as they move down columns or across rows. For example, asked to color every third number beginning with 3 (see fig. 4.28), different students are likely to see different patterns: "Some rows have three and some have four," or "The pattern goes sideways to the left." Some students, seeing the diagonals in the pattern, will no longer count by threes in order to complete the pattern. Teachers need to ask these students to explain to their classmates how they know what to color without counting. Teachers also extend students' mathematical reasoning by posing new questions and asking for arguments to support their answers. "You found patterns when counting by twos, threes, fours, fives, and tens on the hundred board. Do you think there will be patterns if you count by sixes, sevens, eights, or nines? What about counting by elevens or fifteens or by any numbers?" With calculators, students could extend their explorations of these and other numerical patterns beyond 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Fig. **4.28.** Patterns on a hundred board

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Students' reasoning about classification varies during the early years. For instance, when kindergarten students sort shapes, one student may pick up a big triangular shape and say, "This one is big," and then put it with other large shapes. A friend may pick up another big triangular shape, trace its edges, and say, "Three sides—a triangle!" and then put » it with other triangles. Both of these students are focusing on only one property, or attribute. By second grade, however, students are aware that shapes have multiple properties and should suggest ways of classifying that will include multiple properties.

By the end of second grade, students also should use properties to reason about numbers. For example, a teacher might ask, "Which number does not belong and why: 3, 12, 16, 30?" Confronted with this question, a student might argue that 3 does not belong because it is the only single-digit number or is the only odd number. Another student might say that 16 does not belong because "you do not say it when counting by threes." A third student might have yet another idea and state that 30 is the only number "you say when counting by tens."

Students must explain their chains of reasoning in order to see them clearly and use them more effectively; at the same time, teachers should model mathematical language that the students may not yet have connected with their ideas. Consider the following episode, adapted from Andrews (1999, pp. 322–23):

> One student reported to the teacher that he had discovered "that a triangle equals a square." When the teacher asked him to explain, the student went to the block corner and took two half-unit (square) blocks, two half-unit triangular (triangle) blocks, and one unit (rectangle) block (shown in fig. 4.29). He said, "If these two [square half-units] are the same as this one unit and these two [triangular half-units] are the same as this one unit, then this square has to be the same as this triangle!"

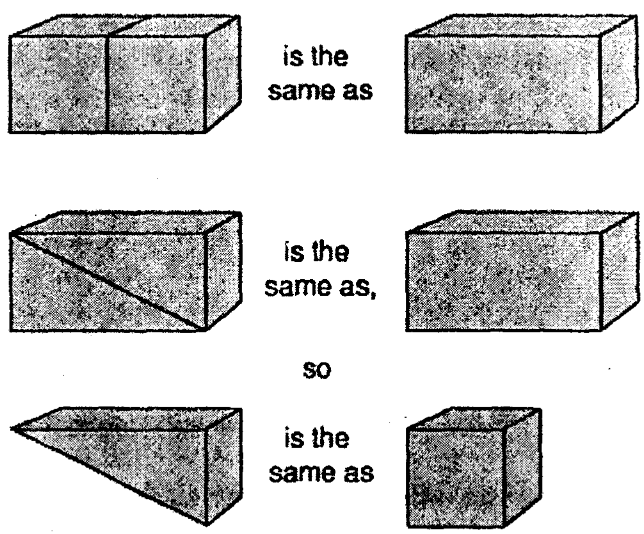

Fig. **4.29.** A student's explanation of the equal areas of square and trianglular block faces

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Even though the student's wording—that shapes were "equal"—was not correct, he was demonstrating powerful reasoning as he used the blocks to justify his idea. In situations such as this, teachers could point to the faces of the two smaller blocks and respond, "You discovered that » the area of this square equals the area of this triangle because each of them is half the area of the same larger rectangle."

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What should be the teacher's role in developing reasoning and proof in prekindergarten through grade 2?

Teachers should create learning environments that help students recognize that all mathematics can and should be understood and that they are expected to understand it. Classrooms at this level should be stocked with physical materials so that students have many opportunities to manipulate objects, identify how they are alike or different, and state generalizations about them. In this environment, students can discover and demonstrate general mathematical truths using specific examples. Depending on the context in which events such as the one illustrated by figure 4.29 take place, teachers might focus on different aspects of students' reasoning and continue conversations with different students in different ways. Rather than restate the student's discovery in more-precise language, a teacher might pose several questions to determine whether the student was thinking about equal areas of the faces of the blocks, or about equal volumes. Often students' responses to inquiries that focus their thinking help them phrase conclusions in more-precise terms and help the teacher decide which line of mathematical content to pursue.

Teachers should prompt students to make and investigate mathematical conjectures by asking questions that encourage them to build on what they already know. In the example of investigating patterns on a hundred board, for instance, teachers could challenge students to consider other ideas and make arguments to support their statements: "If we extended the hundred board by adding more rows until we had a thousand board, how would the skip-counting patterns look?" or "If we made charts with rows of six squares or rows of fifteen squares to count to a hundred, would there be patterns if we skip-counted by twos or fives or by any numbers?"

Through discussion, teachers help students understand the role of nonexamples as well as examples in informal proof, as demonstrated in a study of young students (Carpenter and Levi 1999, p. 8). The students seemed to understand that number sentences like 0 + 5869 = 5869 were always true. The teacher asked them to state a rule. Ann said, "Anything with a zero can be the right answer." Mike offered a counterexample: "No. Because if it was 100 + 100 that's 200." Ann understood that this invalidated her rule, so she rephrased it, "I said, umm, if you have a zero in it, it can't be like 100, because you want just plain zero like 0 + 7 = 7."

The students in the study could form rules on the basis of examples. Many of them demonstrated the understanding that a single example was not enough and that counterexamples could be used to disprove a conjecture. However, most students experienced difficulty in giving justifications other than examples.

p. 125 From the very beginning, students should have experiences that help them develop clear and precise thought processes. This development of reasoning is closely related to students' language development and is dependent on their abilities to explain their reasoning rather than just » give the answer. As students learn language, they acquire basic logic words, including *not, and, or, all, some, if...then,* and *because.* Teachers should help students gain familiarity with the language of logic by using such words frequently. For example, a teacher could say, "You may choose an apple or a banana for your snack" or "If you hurry and put on your jacket, then you will have time to swing." Later, students should use the words modeled for them to describe mathematical situations: "If six green pattern blocks cover a yellow hexagon, then three blues also will cover it, because two greens cover one blue."

Sometimes students reach conclusions that may seem odd to adults, not because their reasoning is faulty, but because they have different underlying beliefs. Teachers can understand students' thinking when they listen carefully to students' explanations. For example, on hearing that he would be "Star of the Week" in half a week, Ben protested, "You can't have half a week." When asked why, Ben said, "Seven can't go into equal parts." Ben had the idea that to divide 7 by 2, there could be two groups of 3, with a remainder of 1, but at that point Ben believed that the number 1 could not be divided.

Teachers should encourage students to make conjectures and to justify their thinking empirically or with reasonable arguments. Most important, teachers need to foster ways of justifying that are within the reach of students, that do not rely on authority, and that gradually incorporate mathematical properties and relationships as the basis for the argument. When students make a discovery or determine a fact, rather than tell them whether it holds for all numbers or if it is correct, the teacher should help the students make that determination themselves.

Teachers should ask such questions as "How do you know it is true?" and should also model ways that students can verify or disprove their conjectures. In this way, students gradually develop the abilities to determine whether an assertion is true, a generalization valid, or an answer correct and to do it on their own instead of depending on the authority of the teacher or the book.

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