Problem Solving Standard for Grades Pre-K–2

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

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Problem solving is a hallmark of mathematical activity and a major means of developing mathematical knowledge. It is finding a way to reach a goal that is not immediately attainable. Problem solving is natural to young children because the world is new to them, and they exhibit curiosity, intelligence, and flexibility as they face new situations. The challenge at this level is to build on children's innate problem-solving inclinations and to preserve and encourage a disposition that values problem solving. Teachers should encourage students to use the new mathematics they are learning to develop a broad range of problem-solving strategies, to pose (formulate) challenging problems, and to learn to monitor and reflect on their own ideas in solving problems.

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What should problem solving look like in prekindergarten through grade 2?

Problem solving in the early years should involve a variety of contexts, from problems related to daily routines to mathematical situations arising from stories. Students in the same classroom are likely to have very different mathematical understandings and skills; the same situation that is a problem for one student may elicit an automatic response from another. For instance, when first-grade students were working in small groups to create models of animals with geometric solids, some had difficulty seeing the parts of animals as geometric shapes. Other students readily saw that they could use seven rectangular prisms to make a giraffe (see figure 4.24). Similarly, the question "How many books would there be on the shelf if Marita put six books on it and Al put three more on it?" may not be a problem for the student who knows the basic number combination 6 and 3 and its connection with the question. For the student who has not yet learned the number combination and may not yet know how to represent the task symbolically, this problem presents an opportunity to learn the skills needed to solve similar problems.

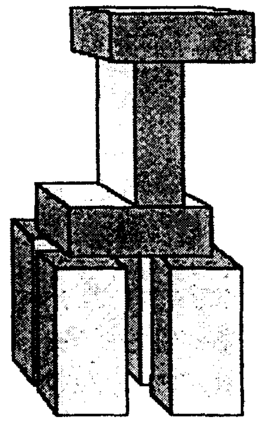

Fig. **4.24.** A block giraffe made from seven rectangular prisms (Adapted from Russell, Clements, and Sarama [1998, p. 115])

Solving problems gives students opportunities to use and extend their knowledge of concepts in each of the Content Standards. For example, many problems relate to classification, shape, or space: Which blocks will fit on this shelf? Will this puzzle piece fit in the space that remains? How are these figures alike and how are they different? In answering these questions, students are using spatial-visualization skills and their knowledge of transformations. Other problems support students' development of number sense and understanding of operations: How many more days until school vacation? There are 43 cards in this group; how many packets of 10 can we make? If there are 26 students in our class and 21 are here today, how many are absent? When young students solve problems that involve comparing and completing collections by using counting strategies, they develop a better understanding of addition and subtraction and the relationship between these operations.

Posing problems, that is, generating new questions in a problem context, is a mathematical disposition that teachers should nurture and develop. Through asking questions and identifying what information is essential, students can organize their thoughts, as the following episode drawn from classroom observations demonstrates:

> Lei wanted to know all the ways to cover the yellow hexagon using pattern blocks. At first she worked with the

blocks using fairly undirected trial and error. Gradually she became more methodical and placed the various arrangements in rows. The teacher showed her a pattern-block program on the class computer and how to "glue" the pattern-block designs together on the screen. Lei organized the arrangements by the numbers of blocks used and began predicting which attempts would be transformations of other arrangements even before she completed the hexagons (see fig. 4.25). The next challenge Lei set for herself was to see if she could create a hexagonal figure using only the orange squares. She had experimented with square blocks and could not make a hexagon. "But," she explained to her teacher, "it might be different on the computer," indicating that she felt the computer was a powerful problem-solving tool.

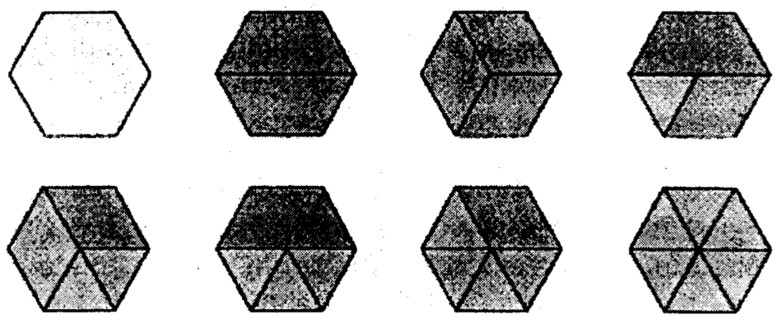

Fig. **4.25.** Organizing arrangements that make hexagons

Kyle was certain that he could find more arrangements for hexagons than Lei had found. Other students joined the discussions between Lei and Kyle. When this activity created a great demand for "turns" with the pattern blocks and the computer, the teacher took advantage of the class's interest by having students discuss how they would know when an arrangement of blocks was a duplicate and how they might keep a written record of their work.

p. 117 Kyle's participation illustrates that students are persistent when problems are interesting and challenging. Their interest also stimulates curiosity in other students. »

Students working together often begin to solve problems one way and, before reaching a solution, change their strategies. In addition, as they create and modify their strategies, students often recognize the need to learn more mathematics. The following episode, drawn from classroom observation, illustrates how teachers can make a problem mathematically rich.

Several first-grade classes in the same school were planting a garden in the school courtyard. The students wanted each class to have the same amount of space for planting; thus, how to divide the area into three equal parts was greatly debated. A walkway, two shade trees, and several benches complicated the discussions. The students began to list all their concerns and the questions they needed to answer before dividing the area for the garden: How big can the garden be? Do the three sections have to be the same shape? How can we be sure each class has the same amount of space?

The teachers drew a large map for each class and indicated the approximate location and amount of space taken by trees, walkway, and benches. In one class the students decided they wanted rectangular gardens and needed to measure the courtyard to figure out how large the rectangles could be. After many measurements and much debate, they cut out three rectangles that were four feet by nine feet to show how big each garden could be. When they were not certain how to use this information on their map, the teacher showed them a scale on a road map and how map scales are used. She suggested appropriate dimensions for the three rectangles, which they cut and glued to their map.

The second class began with a discussion of what "the same area" means. They used large-grid paper squares and taped them to their map to allocate the maximum space for gardening, counting carefully to be certain each class had the same number of squares even though the shapes of the regions were different. This group also needed to learn about scale to actually make a plan for marking off the gardens outside.

Before voting on how to mark off the gardens, the two groups presented their plans to all the classes.

Deciding how to share land for a garden is an example of a classroom-based problem that facilitates students' development of problem-solving strategies. The task was complex. The students struggled with how to share the area equally, how to measure, and how to communicate their ideas. However, the project was rich with proposed strategies, counterproposals, and opportunities for the teachers to introduce new mathematics.

Children's literature is helpful in setting a context for both student-generated and teacher-posed problems. For example, after reading *1 Hunter* (Hutchins 1982) to her class, a second-grade teacher asked students to figure out how many animals, including the hunter, were in the story. Figure 4.26 illustrates several approaches used by the students.

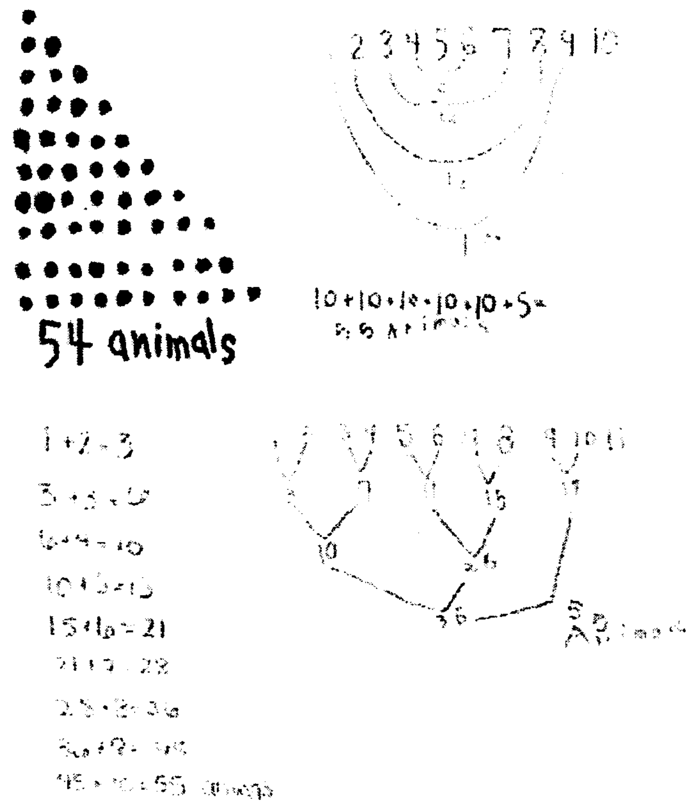

Fig. **4.26.** Detemining the number of animals in *1 Hunter*

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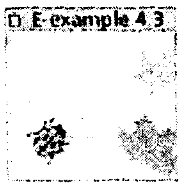
Sharing gives students opportunities to hear new ideas and compare them with their own and to justify their thinking. As students struggle with problems, seeing a variety of successful solutions improves their » chance of learning useful strategies and allows them to determine if some strategies are more flexible and efficient. When the teacher invited the students to explain their solutions to the *1 Hunter* problem, several of them discovered their counting or computational errors and made corrections during the presentations. Explaining their pictorial and written solutions helped them articulate their thinking and make it precise.

Top

What should be the teacher's role in developing problem solving in prekindergarten through grade 2?

The decisions that teachers make about problem-solving opportunities influence the depth and breadth of students' mathematics learning. Teachers must be clear about the mathematics they want their students to accomplish as they structure situations that are both problematic and attainable for a wide range of students. They make important decisions about when to probe, when to give feedback that affirms what is correct and identifies what is incorrect, when to withhold comments and plan similar tasks, and when to use class discussions to advance the students' mathematical thinking. By allowing time for thinking, believing that young students can solve problems, listening carefully to their explanations, and structuring an environment that values the work that students do, teachers promote problem solving and help students make their strategies explicit.



Navigating Paths and Mazes (Part 2)

Instead of teaching problem solving separately, teachers should embed problems in the mathematics-content curriculum. When teachers integrate problem solving into the context of mathematical situations, students recognize the usefulness of strategies. Teachers should choose specific problems because they are likely to prompt particular strategies and allow for the development of certain mathematical ideas. For example, the problem "I have pennies, dimes, and nickels in my pocket. If I take three coins out of my pocket, how much money could I have taken?" can help children learn to think and record their work.

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Tangram Challenges (Part 2)

Assessing students' abilities to solve problems is more difficult than evaluating computational skills. However, it is imperative that teachers » gather evidence in a variety of ways, such as through students' work and conversations, and use that information to plan how to help individual students in a whole-class context. Knowing students' interests allows teachers to formulate problems that extend the mathematical thinking of some students and that also reinforce the concepts learned by other students who have not yet reached the same understandings. Classrooms in which students have ready access to materials such as counters, calculators, and computers and in which they are encouraged to use a wide variety of strategies support thinking that results in multiple levels of understanding.

Two examples illustrate how conversations with students give teachers useful information about students' thinking. Both examples have been drawn from observations of students.

Katie, a kindergarten student, said that her sister in third grade had taught her to multiply. "Give me a problem," she said. The teacher asked, "How much is three times four?" There was a long pause before Katie replied, "Twelve!" When the teacher asked how she knew, Katie responded, "I counted ducks in my head—three groups with four ducks." Katie, while demonstrating an additive understanding of multiplication by counting the ducks in each group, was also exhibiting an interest in, and readiness for, mathematics that is traditionally a focus in the higher grades. Luis, a second grader, demonstrated fluency with composing and decomposing numbers when he announced that he could figure out multiplication. His teacher asked, "Can you tell me four times seven?" Luis was quiet for a few moments, and then he gave the answer twenty-eight. When the teacher asked how he got twenty-eight, Luis replied, "Seven plus three is ten, and four more is fourteen; six more is twenty and one more is twenty-one; seven more is twenty-eight." Luis's approach also built on additive thinking but with a far more sophisticated use of number relationships. He added 7 + 7 + 7 + 7 mentally by breaking the sevens into parts to complete tens along the way.

Students are intrigued with calculators and computers and can be challenged by the mathematics that technology makes available to them, as shown in the following episode, adapted from Riedesel (1980, pp. 74–75):

> Erik, a very capable kindergarten student, observed his teacher using a calculator and asked how it worked. The teacher showed him how to compute simple additions. Erik took the calculator to the math corner and a few minutes later loudly proclaimed, "Five plus four equals nine. Hey, this thing got it right!" A few minutes later, he walked over to the teacher and they had the following conversation:
>
> *Erik:* What does this button mean?
>
> *Teacher:* That's called the "square root." It's a pretty difficult idea in math.
>
> *Erik:* OK. *(He wanders away, but not for long.)* But this is a disaster! I pressed 2, then the square-root key, and I got a whole lot of numbers.
>
> *Teacher:* Try using 1. *(Erik tries this.)*
>
> p. 120 *Erik:* That just gives 1 back. »
>
> *Teacher:* Try 4. *(Erik notes that the result is 2 and asks why. The teacher tells him to get the square tiles and put out one.)* Is that a square? *(Erik nods.)* Try to add more tiles beside this one until it is a square again. *(Erik adds one tile.)* Is that a square?
>
> *Erik:* No, it's a rectangle. *(The teacher asks how he could make it into a square, and Erik adds two more tiles.)*

Teacher: How many tiles are there in all? *(Erik responds that there are four.)* Good. Press 4 on the calculator. How long is the bottom of the square?

Erik: Two.

Teacher: And here at the left side?

Erik: Two there, too.

Teacher: Press the square-root key.

Erik: Hey, it comes out 2!

The teacher challenged Erik to add more tiles until he made another square. Erik built a 3×3 array, counted the total tiles, entered this number into the calculator, and pressed the square-root key. He found that the result was the number of rows and also the number of tiles in each row. Erik kept building squares until at a 9×9 array he said his eyes hurt. The teacher asked him what he had found out.

Erik: Well, if you make a square, then all you have to do is count the tiles and press that number and the square-root key and the calculator tells you how many tiles there are on each side.

Teacher: Good work! What else does that number mean?

Erik: It means that there are that many rows and that many tiles in each row. *(The teacher congratulates Erik on figuring this out.)* Yeah, I guess if you want to learn something really bad, you can. Tomorrow, I'm going to go up to one hundred!

Teachers should ask students to reflect on, explain, and justify their answers so that problem solving both leads to and confirms students' understanding of mathematical concepts. For example, following an estimation activity in a first-grade class, students learned that there were eighty-three marbles in a jar. There were twenty-five students in the class, so the teacher asked how many marbles each child could get. Graham said, "Three." When the teacher asked how he knew, Graham replied, "Eighty-three is just a little more than seventy-five, so we only get three. There are four quarters in a dollar. There are three quarters in seventy-five cents. So we can only get three."

Teachers must make certain that problem solving is not reserved for older students or those who have "got the basics."

Young students can engage in substantive problem solving and in doing so develop basic skills, higher-order-thinking skills, and problem-solving strategies (Cobb et al. 1991; Trafton and Hartman 1997).

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